

# **Statistics of Credit Risk Management in Financial Derivatives**

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## Overview

- derivatives
- credit risk
- risk mitigation
- use of collateral
- intermediation
- capital adequacy
- portfolio management

## Derivatives:

“A financial instrument whose value *derives from* some underlying asset”

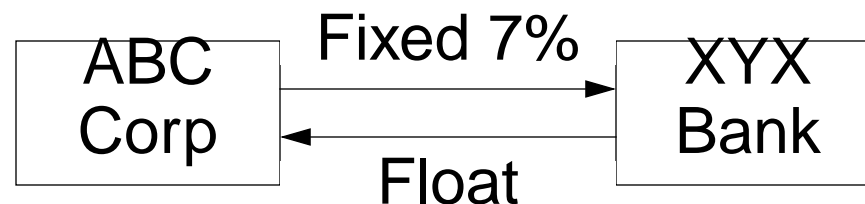
Futures, options, etc.

*Over-the-counter* derivatives contract:

- two-party agreement to exchange cash flows;
- lasts for a specified period of time;
- cash flows determined by future market rates, e.g. interest rates.

## Example: Interest rate swap

- ABC Corp agrees to pay to XYZ Bank a fixed interest rate of 7% on a principal amount of \$10,000,000 for a term of 5 years.
- XYZ Bank agrees to pay to ABC Corp a “floating” interest rate<sup>1</sup> on the same principal amount for the same term.



1. E.g. LIBOR (London InterBank Offered Rate), currently 5.97% for 6-month deposits, + a spread.

## Example: Interest rate swap (contd.)

- “Principal amount” does not change hands.
- Only a *net* payment is made on any payment date.

## Marking to Market

Mark-to-Market value:

- net present value (NPV) of remaining cash flows.

Mark-to-market value changes over time:

- with market rates;
- as deal matures.

E.g. If rates rise, ABC expects to receive net payments from XYZ. ABC is “in the money”.

## Credit Risk

### Default Risk:

- one party may fail to honor contract;
- standard contract requires prompt settlement of NPV (of this and all other contracts);
- other party is an unsecured creditor, unless security is provided.

## How much risk?

Party A loses money if:

- Party B defaults, and
- A is *in the money*, and
- recovery is less than 100%

## One-year transition probabilities (%):

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	88.72	8.14	0.66	0.06	0.12	0.00	0.00	0.00	2.29
AA	0.68	88.31	7.59	0.62	0.06	0.14	0.02	0.00	2.58
A	0.09	2.19	87.74	5.32	0.71	0.25	0.01	0.06	3.64
BBB	0.02	0.31	5.61	81.95	5.00	1.10	0.11	0.18	5.72
BB	0.03	0.13	0.61	7.03	73.27	8.04	0.91	1.06	8.93
B	0.00	0.10	0.21	0.38	5.66	72.91	3.56	5.20	11.98
CCC	0.18	0.00	0.18	1.07	1.96	9.27	53.48	19.79	14.08
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
NR	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

## Cumulative default rates (% , Markov model):

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Year:	1	2	3	4	5	6	7	8	9	10
<b>AAA</b>	0.00	0.00	0.01	0.02	0.03	0.06	0.08	0.12	0.16	0.21
<b>AA</b>	0.00	0.02	0.05	0.10	0.16	0.23	0.32	0.42	0.53	0.65
<b>A</b>	0.06	0.14	0.26	0.40	0.56	0.75	0.96	1.18	1.41	1.66
<b>BBB</b>	0.18	0.46	0.83	1.26	1.74	2.24	2.74	3.25	3.74	4.21
<b>BB</b>	1.06	2.45	3.95	5.44	6.85	8.14	9.29	10.30	11.19	11.96
<b>B</b>	5.20	9.76	13.55	16.64	19.10	21.07	22.63	23.88	24.87	25.66
<b>CCC</b>	19.79	30.88	37.26	41.06	43.41	44.93	45.96	46.68	47.21	47.60

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## Historical multi-year rates (%):

(Generally higher)

Year:	1	2	3	4	5	6	7	8	9	10
<b>AAA</b>	0.00	0.00	0.07	0.15	0.24	0.43	0.66	1.05	1.21	1.40
<b>AA</b>	0.00	0.02	0.12	0.25	0.43	0.66	0.89	1.06	1.17	1.29
<b>A</b>	0.06	0.16	0.27	0.44	0.67	0.88	1.12	1.42	1.77	2.17
<b>BBB</b>	0.18	0.44	0.72	1.27	1.78	2.38	2.99	3.52	3.94	4.34
<b>BB</b>	1.06	3.48	6.12	8.68	10.97	13.24	14.46	15.65	16.81	17.73
<b>B</b>	5.20	11.00	15.95	19.40	21.88	23.63	25.14	26.57	27.74	29.02
<b>CCC</b>	19.79	26.92	31.63	35.97	40.15	41.61	42.64	43.07	44.20	45.10

## Mitigating Risk: Collateral

Party A is in the money.

Party A requests collateral (cash or Treasury securities) from Party B.

How much?

- Market value of collateral =  $C$ ;
- B defaults at most  $d$  days later;
- Want  $P(C + \delta C > NPV + \delta NPV) > 1 - \varepsilon$ .

## Mitigating Risk: Collateral (contd.)

Must:

- specify  $d$  and  $\varepsilon$ ;
- find distribution of  $\delta NPV - \delta C = \delta(NPV - C)$

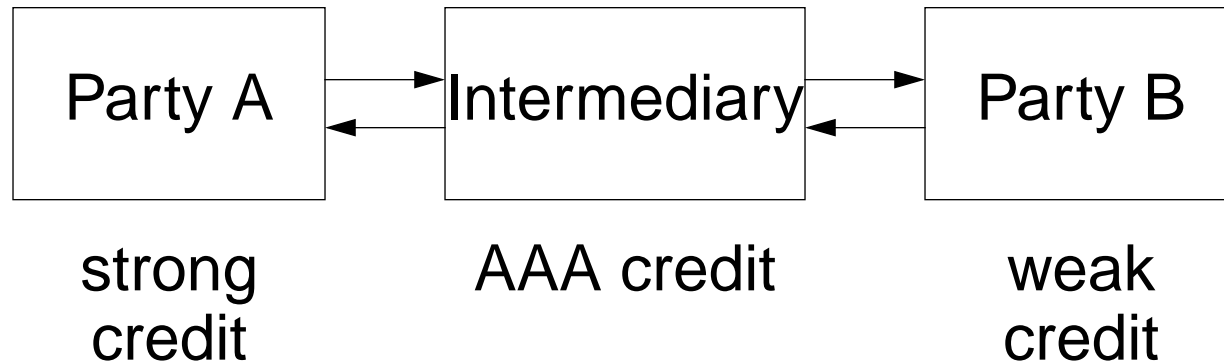
Typically:

- assume normal distribution
- $\mu = 0$ ,  $\sigma =$  calculated SD of  $\delta(NPV - C)$
- set  $C = NPV + k\sigma$ ,  $k = 2$  or  $3$ .

Often SD of  $\delta(NPV - C)$  is replaced by

$$\text{SD of } \delta(NPV) + \text{SD of } \delta(C)$$

## Mitigating Risk: Intermediation



### Possible Intermediaries:

- third party;
- Credit-enhanced Derivative Product Company (“DPC”) wholly owned by Party B.

## Credit Enhancement

How does DPC rate higher than B?

- bankruptcy-remote;
- no market risk;
- no credit exposure to B (collateral);
- enough capital to withstand other credit losses.

## DPC Structures

### Continuation Vehicle:

- survives insolvency of parent;
- Contingent Manager steps in;
- contracts run to maturity.

### Termination Vehicle:

- all contracts terminate on insolvency of parent;
- NPV exchanged promptly with all counterparties.

## Capital Adequacy

Rating agencies require

$$P(\text{credit losses} > \text{capital}) > 1 - \varepsilon$$

or similar.

$$\text{Overall credit loss} = \sum \text{Counterparty loss}$$

$$\text{Counterparty loss} = \max\{0, \text{NPV}(\tau)\}$$

$\tau$  = time of counterparty event

## Capital Adequacy

Assumptions:

1.  $NPV(t)$ -

- depends on market rates at  $t$ ;
- not independent across counterparties.

2.  $\tau$  –

- need joint distribution;
- $\tau = \min\{\tau(\text{counterparty}), \tau(\text{sovereign})\}$
- independence plausible (perhaps conditionally on market conditions).

## Capital Adequacy

Computational strategy:

- condition on market path;
- conditional distribution of credit loss depends on distribution of  $\tau$ 's (and path);
- conditional independence  $\Rightarrow$  distribution of sum is convolution;
- get unconditional distribution by averaging over market paths (simulation);

## Optimization

Capital and collateral cost money.

Required levels of both depend on portfolio of contracts:

- credit ratings of counterparties;
- sensitivity of NPV's to market fluctuations.

Portfolio can be managed to control costs.

## Summary

Modern quantitative methods play a role in:

- mitigating credit risk, either through use of collateral or through structuring a special purpose intermediary;
- efficiently managing a portfolio of derivatives.