

ST 432 Spring 2008
Homework Set 10
Due Tuesday April 8

Q1. Illustrate the point that systematic random sampling is related to cluster sampling for the case where we have a population of $N = 8000$ sampling units and we want to take a systematic random sample of size $n = 400$. Lay out conceptually all the clusters and their individual elements.

Q2. Consider a small artificial population with increasing order with $N = 6$ and the elements $y_1 = 1, y_2 = 2, y_3 = 3, y_4 = 4, y_5 = 5,$ and $y_6 = 6$.

Consider all the systematic samples of size $n = 2$ and show $E(\bar{y}_{sys}) = 3.5$ an unbiased estimate, and $Var(\bar{y}_{sys}) = 0.667$.

Also consider all the simple random samples of size $n = 2$ and hence show directly that in this case systematic sampling is better (i.e., more efficient).

Q3. Consider also a small artificial population with periodicity with $N = 12$ with the values $y_1 = 1, y_2 = 2, y_3 = 3, y_4 = 1, y_5 = 2, y_6 = 3, y_7 = 1, y_8 = 2, y_9 = 3, y_{10} = 1, y_{11} = 2,$ and $y_{12} = 3$.

Consider all systematic samples of size $n = 4$ and show that \bar{y}_{sys} is unbiased. Find $Var(\bar{y}_{sys})$ directly.

Also show that the apparent variance is 0, if one pretends that the systematic random sample is a simple random sample.

Verify that it would be much better in this case to have really used a simple random sample of size 4. Show that $Var(\bar{y}) = 0.12$ compared to $Var(\bar{y}_{sys}) = 0.667$. Hint use the appropriate equation for srs don't do by enumeration of all the srs as it will take too long.