

Common Discrete Probability Distributions

Distribution of rv Y	Probability Mass Function	Mean $E(Y)$	Variance $V(Y)$	mgf $m_Y(t)$
Binomial	$p_Y(y) = \binom{n}{y} p^y (1-p)^{n-y}, \quad y = 0, 1, \dots, n$ $0 < p < 1$	np	$np(1-p)$	$(pe^t + (1-p))^n$
Geometric	$p_Y(y) = p(1-p)^{y-1}, \quad y = 1, 2, \dots$ $0 < p < 1$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p_Y(y) = \frac{\binom{r}{y} \binom{N-r}{n-y}}{\binom{N}{n}},$ $\max(0, n+r-N) \leq y \leq \min(r, n)$ $N, r, n \text{ positive integers, } N \geq r, \text{ and } N \geq n$	$n \frac{r}{N}$	$n \left(\frac{r}{N} \right) \left(1 - \frac{r}{N} \right) \left(\frac{N-n}{N-1} \right)$	Closed form for $m_Y(t)$ does not exist.
Poisson	$p_Y(y) = \frac{e^{-\lambda} \lambda^y}{y!}, \quad y = 0, 1, \dots$ $\lambda > 0$	λ	λ	$e^{[\lambda(e^t - 1)]}$
Negative Binomial	$p_Y(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}, \quad y = r, r+1, \dots$ $0 < p < 1, \quad r = 1, 2, \dots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{pe^t}{1-(1-p)e^t} \right)^r$