

ST745, Spring 2005

Homework 3, due: Tuesday, 2/15/2005

♡ Happy Valentine's Day ♡

1. (5 pts) Suppose we have a small data set with different kinds of censoring: 2, 3, 4⁺, 5⁻, 6, 7⁺, [8,9], where ⁺ (⁻) means right (left) censored observations and [a, b] means an interval censored observation. Suppose the distribution of the underlying survival time is an exponential distribution with a constant hazard λ . Write down the likelihood function of λ for this given data set.

2. (10 pts) Fit a Weibull model to the censored survival data in problem 3 of HW2. Let φ be the population median such that $S(\varphi) = 1/2$.

(a) Estimate φ .

(b) Define $\gamma = \log(\varphi)$. Show that γ can be written as

$$\gamma = \sigma \log(\log 2) + \beta_0$$

using the parametrization of Proc Lifereg of SAS.

(c) Estimate the variance of $\hat{\gamma} = \log(\hat{\varphi})$ and hence construct a 95% confidence interval for γ (You need to obtain the estimate of the variance matrix of $(\hat{\beta}_0, \hat{\sigma})$).

(d) Use the confidence interval obtained in (c) for γ to construct a 95% confidence interval for the true median φ .

3. (10 pts) In class, we considered a score test for testing whether or not the survival times are from an exponential distribution under the assumption that they are from a Weibull distribution and used the complete data in HW2 as an illustration. In this problem, you are asked to work with the censored survival data in problem 3 of HW2.

(a) Write down the log-likelihood function of the model parameters assuming the survival data are from a Weibull distribution. Assume the observed data is $(x_i, \delta_i), i = 1, 2, \dots, n$.

(b) Find the score and information matrix from this model and then evaluate them under the hypothesis that the data are from an exponential distribution.

(c) Use the censored survival data in HW2, perform the score test to test whether or not the survival times are from an exponential distribution.

4. (10 pts) Using `proc lifereg` in SAS with the censored survival data in problem 3 of HW2, do the following:
 - (a) Perform Wald test to test whether or not the survival times are from an exponential distribution.
 - (b) Perform likelihood ratio test to test whether or not the survival times are from an exponential distribution.

5. (5 pts) The above problems all are under the assumption that the data are from a Weibull model. Suggest ways to check this model assumption and conduct the diagnostics.