

ST745, Spring 2005

Homework 1, due: Thursday, 1/20/2005

1. (10pts) Assume the random survival time (in years) T has survival function $S(t) = 64/(t + 8)^2$.

Do the following:

- (a) Find the mean and median survival times.
- (b) Find the hazard function of T .
- (c) Find the mortality rate $m(t)$ at time t .
- (d) Find the average remaining survival time after $t_0 = 8$. How does it compare to the mean survival time you got in (a)? How do you explain this?

2. (10 pts) In class we claimed that the sample mean of censored survival times is no longer an unbiased estimator of the population mean. Let us prove this claim for a special case. Suppose the life time T (in days) of some light bulbs has an exponential distribution with constant hazard λ and we are interested in estimating the mean lifetime $\mu_T = 1/\lambda$. We picked a random sample of the light bulbs and plan to test them for L (e.g., 30) days. So for those light bulbs which break before L days, we will observe their actual life times. For those bulbs which is still working after L days, we don't know their life times and we only know that their life times are greater than L . That is, we will have a random sample of the random variable $X = \min(T, L)$. Do the following:

- (a) Find the survival function of X . That is, find $P[X \geq t]$ for any t (**Hint**: consider two cases: $t \geq L$ and $t < L$).
- (b) Find $E(X)$ using the survival function of X you got in (a) and the formula $E(X) = \int_0^\infty S_X(t)dt$, where $S_X(t)$ is the survival function you got in (a).
- (c) What is your conclusion on using sample mean to estimate the population mean when censoring is present based on $E(X)$ and $E(T)$?

3. (10 pts) The time in days to development of a tumor for rats exposed to a carcinogen follows a Weibull distribution with $\alpha = 2$ and $\lambda = 0.002$.

- (a) Find the probabilities that a (random) rat will be tumor free at 10 days, 20 days and 30 days.
- (b) What is the average time to tumor development? (Hint: $\Gamma(0.5) = \sqrt{\pi}$, where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$)
- (c) Find the hazard rate of time to tumor development at 10 days, 20 days and 30 days.
- (d) Find the median time to tumor development.

4. (10 pts) Suppose the hazard function of a random survival time T is given by

$$\lambda(t) = \begin{cases} \lambda_1 & 0 = t_0 \leq t < t_1 \\ \lambda_2 & t_1 \leq t < t_2 \\ \lambda_3 & t_2 \leq t < \infty \end{cases}$$

- (a) Find the survival function for this model.
- (b) Using your favorite software plot the survival function in $[0, 100]$ for the special case where $t_1 = 10$, $t_2 = 30$, $t_3 = \infty$ and $\lambda_1 = 0.01$, $\lambda_2 = 0.03$, $\lambda_3 = 0.02$. Find the average survival time and median survival time for this model. Assume the time unit is year. (**Hint:** An R example is available in the class website)